Analysis and correction of far-field diffraction pattern for corner-cube reflector

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RESEARCH ARTICLE

Analysis and correction of far-field diffraction pattern for corner-cube reflector

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Abstract The six different polarization transformation matrices emerging from the six paths through a corner cube reflector will worsen the far-field diffraction pattern. In this study, the polarization transformation matrices can be obtained by polarization ray tracing. Then substituting the polarization transformation matrices into the diffraction integral, the central intensity can be got. Analyzing the central intensity, the approach to make the polarization transformation matrices the same is coating at the rear of the three totally reflection surfaces and making the $\left|\delta_p - \delta_s\right| = \pi$ in each of the three surfaces. Thus, the far-field diffraction pattern is always 'perfect' Airy pattern no matter what the polarization state of the incident light. We also give the design of thin film to make $\left|\delta_p - \delta_s\right| = \pi$.

Keywords Polarization · Far-field diffraction pattern · Phase coating · CCR

Introduction

Corner cube reflectors (CCRs) are commonly used in interferometers, surveying references, gravimeters, and for laser ranging to satellites and the Moon [1] because of their geometrically simple structure, and their polarization properties have been the subject of particular interest. Liu and Azzam [2] offer a discussion about the polarization property emerging from total internal reflection (TIR) CCRs by use of ray tracing, along with laboratory measurements of Stokes parameters. TIR corner cubes, generally will introduce six different elliptical polarization states. And these can be changed by coating in the three TIR surfaces. Bieg [3] provides a calculation in the polarization properties of a metal CCR. The different elliptical polarization states emerging from the six unique paths through the corner cube complicate the far-field diffraction pattern by introducing various phase delays between the six paths. One of the efficient methods for changing and optimizing the far field diffraction pattern is to control of the phase shift of the reflection coefficient at the CCR faces, which is determined by the kind of the face coating or its absence. Sadovnikov and Sokolov [4], Sokolov and Murashkin [5] and Murphy and Goodrow [1], presenting diagrams of polarization and diffraction patterns at different input polarizations for the normal incidence case. However, these works not made a solution to make the polarization transform matrices the same and to obtain a perfect Airy pattern.

In this paper, the special condition for the ideal CCR, which makes the far-field diffraction pattern close to a Airy pattern no matter what the polarization state, is obtained. And according to the special condition, we design a thin film system, which likes AR coating but the thickness of each layer needs to change.

Calculation of polarization property

To simplify ray tracing, the corner cube is represented as shown in Fig. 1. The three right-angle isosceles triangles *ABO*, *ACO*, and *CBO* are the internal reflecting surfaces of the corner cube, and are located in the *XY*, *XZ*, and *YZ* planes, respectively. Equilateral triangle *ABC* is the base. The unit inner normal vectors of the four surfaces are:

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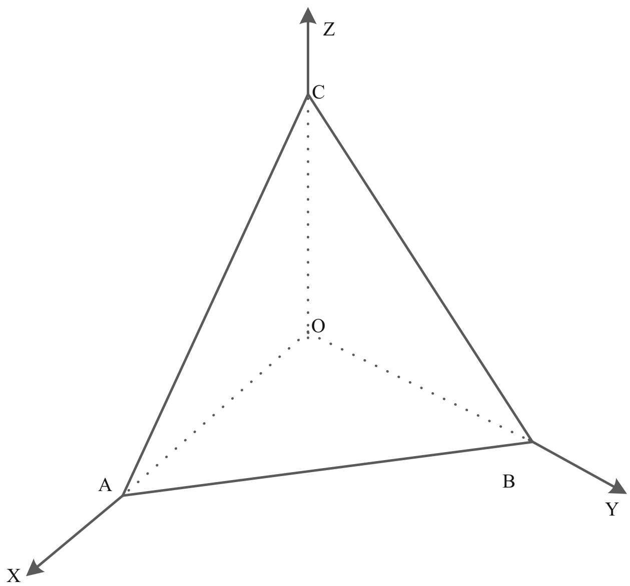


Fig. 1 Geometry of the corner-cube reflector

Surface CBO: $\vec{N}_1 = \vec{i}$
Surface ACO: $\vec{N}_2 = \vec{j}$
Surface ABO: $\vec{N}_3 = \vec{k}$
Surface ABC (outer): $\vec{N}_4 = \frac{\sqrt{3}}{3}\left(\vec{i} + \vec{j} + \vec{k}\right)$

We assume that a normally incident input ray which direction opposite to $\vec{N}_4$, and is reflected totally from the internal surfaces in the order of CBO, ACO, and ABO, and output from the surface ABC which direction parallel to N4. The unit vectors along the incident and reflected light from each internal surface are related by:

$$\vec{A'} = \vec{A} - 2\vec{N}\left(\vec{A} \cdot \vec{N}\right) \tag{1}$$

where $\vec{A}'$ is the unit incident ray vector along the incident ray, $\vec{A}$ is the unit vector along the reflected ray and $\vec{N}$ is the unit normal vector of the surface.

It is not normal incidence on surfaces CBO, ACO and ABO, so the Fresnel formulas can be used in the local kps coordinate system. The reflection Jones matrix in i-th surface is taken as in the following form:

$$R_j = \begin{bmatrix} r_p \exp(i\delta_j) & 0 \\ 0 & r_s \exp(-i\delta_j) \end{bmatrix} \tag{2}$$

where subscript 1 represents surface CBO; 2, ACO; 3, ABO and 0, ABC.

The k-p-s frame is described by (k is the unit wave vector, which describes in the following $\overrightarrow{A}$):

$$\overrightarrow{S_j} = \overrightarrow{S_j'} = \frac{\overrightarrow{A_j} \times \overrightarrow{A_j'}}{\left|\overrightarrow{A_j} \times \overrightarrow{A_j'}\right|}, \quad \overrightarrow{P_j} = \overrightarrow{S_j} \times \overrightarrow{A_j}, \quad \overrightarrow{P_j'} = \overrightarrow{S_j'} \times \overrightarrow{A_j'} \tag{3}$$

In the matrix, $r_{jp} = r_{js} = 1$ because of the total reflection, and $\delta_j = \frac{\delta_{jp} - \delta_{js}}{2}$ is half of the difference between the reflection phase shift of parallel δ_{jp} and vertical δ_{js} on the j th surface, which can be calculated by the use of total refelection theory; And i is imaginary unit.

The Jones vectors are different in two planes when the two planes not coplanes. It is need to rotate the coordinate system when the light leaves from the i-th surface and incidents on the (i + 1) th surface. Well, the rotation matrix is defined as:

$$C_{i,i+1}(\beta_i) = \begin{bmatrix} \overrightarrow{P_{i+1}} \cdot \overrightarrow{P_i'} & \overrightarrow{P_{i+1}} \cdot \overrightarrow{S_i'} \\ \overrightarrow{S_{i+1}} \cdot \overrightarrow{P_i'} & \overrightarrow{S_{i+1}} \cdot \overrightarrow{S_i'} \end{bmatrix} = \begin{bmatrix} \cos(\beta_i) & \sin(\beta_i) \\ -\sin(\beta_i) & \cos(\beta_i) \end{bmatrix} \tag{4}$$

where β_i is the angle of rotation of coordinates. Thus, the form of the overall Jones matrix in one path can be written as:

$$J_{123} = C_{3,0}(\beta_1)R_3C_{2,3}(\beta_2)R_2C_{1,2}(\beta_3)R_1C_{0,1}(\beta_4) \tag{5}$$

The phase shift can be changed by coating on the surfaces CBO, ACO and ABO when describing the reflection, so the phase shift can be a variable δ_i. According to our calculation, the angles of the incidence on the three surfaces are the same, so the phase shifts are also the same when coating the same thin film system. That is $\delta_1 = \delta_2 = \delta_3 = \delta$ and $\delta = 22.6^\circ$ when the coating is absent for the refractive index of CCR is 1.52.

The polarization transform matrices of the six paths for the corner-cube reflector can be described listing in Table 1 (the subscript notation is already explained above):where

$$J_{123} = C_{3,0}\left(-120^\circ\right)R_3C_{2,3}\left(60^\circ\right)R_2C_{1,2}\left(-60^\circ\right)R_1C_{0,1}\left(120^\circ\right)$$
$$J_{132} = C_{3,0}\left(0^\circ\right)R_3C_{2,3}\left(-60^\circ\right)R_2C_{1,2}\left(60^\circ\right)R_1C_{0,1}\left(120^\circ\right)$$
$$J_{213} = C_{3,0}\left(-120^\circ\right)R_3C_{2,3}\left(-60^\circ\right)R_2C_{1,2}\left(60^\circ\right)R_1C_{0,1}\left(0^\circ\right)$$
$$J_{231} = C_{3,0}\left(120^\circ\right)R_3C_{2,3}\left(60^\circ\right)R_2C_{1,2}\left(-60^\circ\right)R_1C_{0,1}\left(0^\circ\right)$$
$$J_{312} = C_{3,0}\left(0^\circ\right)R_3C_{2,3}\left(60^\circ\right)R_2C_{1,2}\left(-60^\circ\right)R_1C_{0,1}\left(-120^\circ\right)$$
$$J_{321} = C_{3,0}\left(120^\circ\right)R_3C_{2,3}\left(-60^\circ\right)R_2C_{1,2}\left(60^\circ\right)R_1C_{0,1}\left(-120^\circ\right)$$

Comparing our result with literature [2], it is found that only the first and the last matrices are not the same. That is because the choice of kps frame is not the same.

In the generalized coordinates, p and s, the unit electric field vector follows:

$$E_{in} = \begin{bmatrix} E_{pi} \\ E_{si} \end{bmatrix} \tag{6}$$

and setting $\left|E_{pi}\right|^2 + \left|E_{si}\right|^2 = 1$.Then the output electric field vectors, also called output polarization states in optics, are:

Table 1 Jones matrices for six trips of CCR

Form of trips	Jones matrix	Matrix without coating
J_{123}	$\begin{bmatrix} \frac{1}{16}\exp(-3i\delta)(3+15\exp(2i\delta)-3\exp(4i\delta)+\exp(6i\delta)) & -\frac{\sqrt{3}}{16}\exp(-3i\delta)(-1+\exp(2i\delta))(1+\exp(2i\delta))^2 \\ -\frac{\sqrt{3}}{16}\exp(-3i\delta)(-1+\exp(2i\delta))(1+\exp(2i\delta))^2 & \frac{1}{16}\exp(-3i\delta)(1-3\exp(2i\delta)+15\exp(4i\delta)+3\exp(6i\delta)) \end{bmatrix}$	$\begin{bmatrix} 0.7869-0.5481i & -0.2837i \\ -0.2837i & 0.7869+0.5481i \end{bmatrix}$
J_{132}	$\begin{bmatrix} -\frac{1}{8}\exp(-i\delta)(-3+6\exp(2i\delta)+\exp(4i\delta)) & \frac{\sqrt{3}}{8}\exp(-i\delta)(1+\exp(2i\delta))^2 \\ -\frac{\sqrt{3}}{8}\exp(-3i\delta)(1+\exp(2i\delta))^2 & \frac{1}{8}\exp(-3i\delta)(-1-6\exp(2i\delta)+3\exp(4i\delta)) \end{bmatrix}$	$\begin{bmatrix} -0.3934-0.5481i & 0.6814+0.2837i \\ -0.6814+0.2837i & -0.3934+0.5481i \end{bmatrix}$
J_{213}	$\begin{bmatrix} -\frac{1}{8}\exp(-i\delta)(-3+6\exp(2i\delta)+\exp(4i\delta)) & -\frac{\sqrt{3}}{8}\exp(-3i\delta)(1+\exp(2i\delta))^2 \\ \frac{\sqrt{3}}{8}\exp(-i\delta)(1+\exp(2i\delta))^2 & \frac{1}{8}\exp(-3i\delta)(-1-6\exp(2i\delta)+3\exp(4i\delta)) \end{bmatrix}$	$\begin{bmatrix} -0.3934-0.5481i & -0.6814+0.2837i \\ 0.6814+0.2837i & -0.3934+0.5481i \end{bmatrix}$
J_{231}	$\begin{bmatrix} -\frac{1}{8}\exp(-i\delta)(-3+6\exp(2i\delta)+\exp(4i\delta)) & \frac{\sqrt{3}}{8}\exp(-3i\delta)(1+\exp(2i\delta))^2 \\ -\frac{\sqrt{3}}{8}\exp(-i\delta)(1+\exp(2i\delta))^2 & \frac{1}{8}\exp(-3i\delta)(-1-6\exp(2i\delta)+3\exp(4i\delta)) \end{bmatrix}$	$\begin{bmatrix} -0.3934-0.5481i & 0.6814-0.2837i \\ -0.6814-0.2837i & -0.3934+0.5481i \end{bmatrix}$
J_{312}	$\begin{bmatrix} -\frac{1}{8}\exp(-i\delta)(-3+6\exp(2i\delta)+\exp(4i\delta)) & -\frac{\sqrt{3}}{8}\exp(-i\delta)(1+\exp(2i\delta))^2 \\ \frac{\sqrt{3}}{8}\exp(-3i\delta)(1+\exp(2i\delta))^2 & \frac{1}{8}\exp(-3i\delta)(-1-6\exp(2i\delta)+3\exp(4i\delta)) \end{bmatrix}$	$\begin{bmatrix} -0.3934-0.5481i & -0.6814-0.2837i \\ 0.6814-0.2837i & -0.3934+0.5481i \end{bmatrix}$
J_{321}	$\begin{bmatrix} \frac{1}{16}\exp(-3i\delta)(3+15\exp(2i\delta)-3\exp(4i\delta)+\exp(6i\delta)) & \frac{\sqrt{3}}{16}\exp(-3i\delta)(-1+\exp(2i\delta))(1+\exp(2i\delta))^2 \\ \frac{\sqrt{3}}{16}\exp(-3i\delta)(-1+\exp(2i\delta))(1+\exp(2i\delta))^2 & \frac{1}{16}\exp(-3i\delta)(1-3\exp(2i\delta)+15\exp(4i\delta)+3\exp(6i\delta)) \end{bmatrix}$	$\begin{bmatrix} 0.7869-0.5481i & 0.2837i \\ 0.2837i & 0.7869+0.5481i \end{bmatrix}$

$$\begin{aligned} B_1 &= J_{123}E_{in} \\ B_2 &= J_{132}E_{in} \\ B_3 &= J_{213}E_{in} \\ B_4 &= J_{231}E_{in} \\ B_5 &= J_{312}E_{in} \\ B_6 &= J_{321}E_{in} \end{aligned} \tag{7}$$

Figure 2 shows output polarization states at normal incidence for linear input polarization with 0°, 45°and 90°. And Fig. 3 shows output polarization states at normal incidence for left and right circular input polarization.

Correction of far-field diffraction

In a number of cases (geostationary statellites, the Moon) the retroreflector system must produce the Airy diffraction pattern in the far field of the reflected radiation. Now consider a circular aperture with six kinds of polarizations. Then the far-field diffraction integral should become:

$$\overrightarrow{u'}(r,\omega) = \frac{\exp\left[jk\left(f + \frac{r^2}{2f}\right)\right]}{j\lambda f} \sum_{n=1}^{6} B_n \int_0^a \int_{\frac{n-1}{3}\pi}^{\frac{n}{3}\pi} \exp\left[-j\frac{k}{f}\rho r\cos(\omega - \beta)\right]\rho d\rho d\beta \tag{8}$$

where $\overrightarrow{u'}(r,\omega)$ is the amplitude vector at the far field point whose polar coordinate is (r,ω), (ρ,β) is the polar coordinate at the aperture, B_n describes the polarization state of light emerging from the n-th sextant, and $k = \frac{2\pi}{\lambda}$, and a is the radius of aperture.Since our interest is in the distribution of energy, we can normal the amplitude in that:

$$\vec{u}(r,\omega) = \frac{1}{\pi a^2}\sum_{n=1}^{6} B_n \int_0^a \int_{\frac{n-1}{3}\pi}^{\frac{n}{3}\pi} \exp\left[-j\frac{2\pi}{\lambda f}\rho r\cos(\omega - \beta)\right]\rho d\rho d\beta \tag{9}$$

Because the electrical filed has two orthogonal components, while orthogonal polarizations cannot interfere with each other, so each of the outgoing polarization can be treated separately. Then, we can get the intensity distribution by summing the intensity for the x-component and

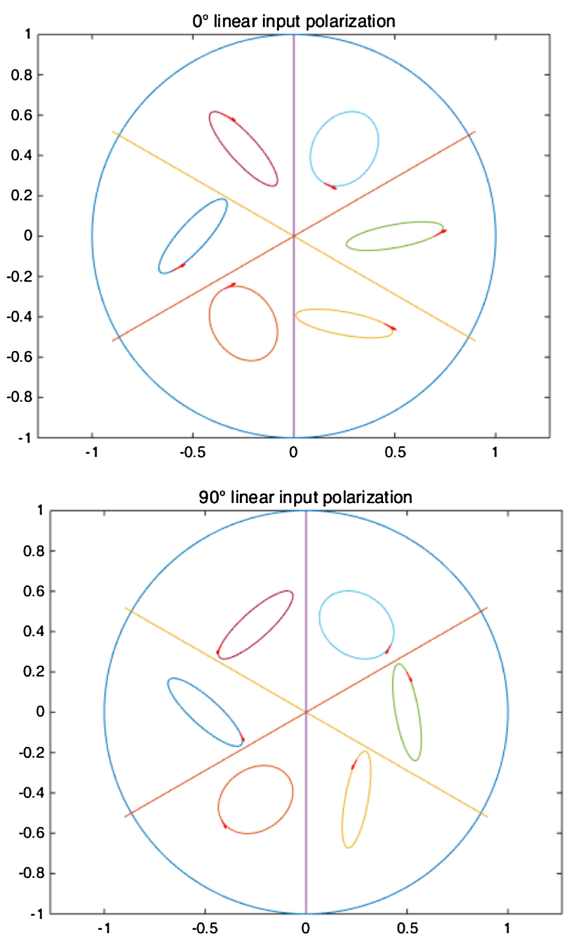


Fig. 2 output polarization states for linear input polarization

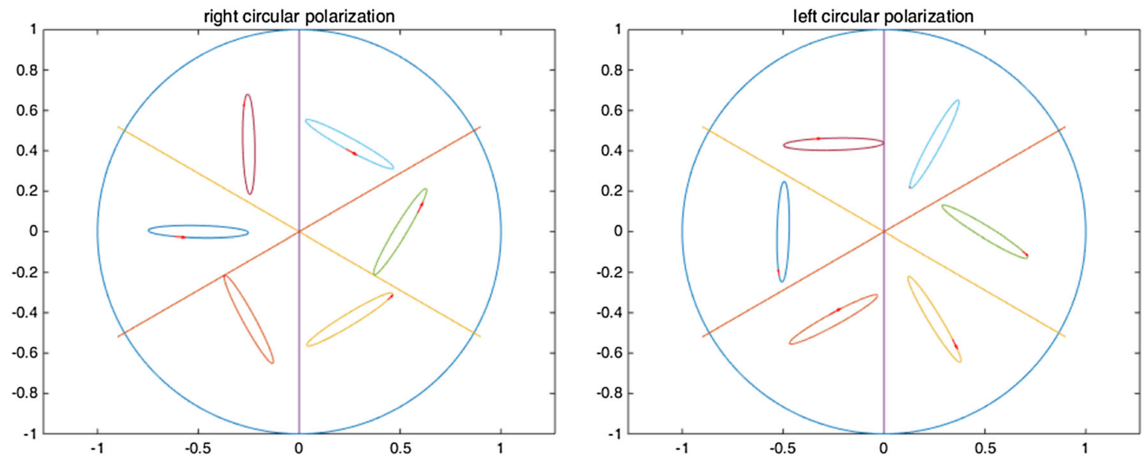


Fig. 3 Output polarization states for circular input polarization

the y-component. Refs. [6] and [1] have described the far field diffraction patterns for different phase shifts and different linear polarization states for normal incidence.

One can readily compute the central intensity I(0,0), of the far-field diffraction pattern in the normal incidence case with the following formula:

$$\begin{aligned} I(0,0) &= \frac{1}{36}\sum_{n=1}^{6} B_n^{\dagger} B_n \\ &= \frac{1}{4}\sin^2(\delta)\left[\sin^2(\delta) - 3\right]^2 \end{aligned} \tag{10}$$

Figure 4 shows the central intensity varying with the half of phase shift δ.

The diffraction patterns for three orientations of linear input polarization 0°, 45°, 90° without coating thin film shown in Fig. 5. It is obvious that these diffraction patterns are worsen.

By analyzing the expression of central intensity, we can get that the maximum value of the central intensity can be get at the point $\delta_p - \delta_s = \pm\pi$. Substituting the condition $\delta_p - \delta_s = \pm\pi$ into the six polarization transform matrices, the six same matrices will get and they all are $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. And when we substitute the condition into the Ref. [2], the same matrices $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are got.

And now substituting the six same polarization transform matrices into the far field diffraction integral, we can get an Airy pattern produced by the CCR no matter what the polarization state of the incidence, shown in many optical books. The diffraction pattern with the thin film satisfying $\delta_p - \delta_s = \pm\pi$ is shown in Fig. 6.

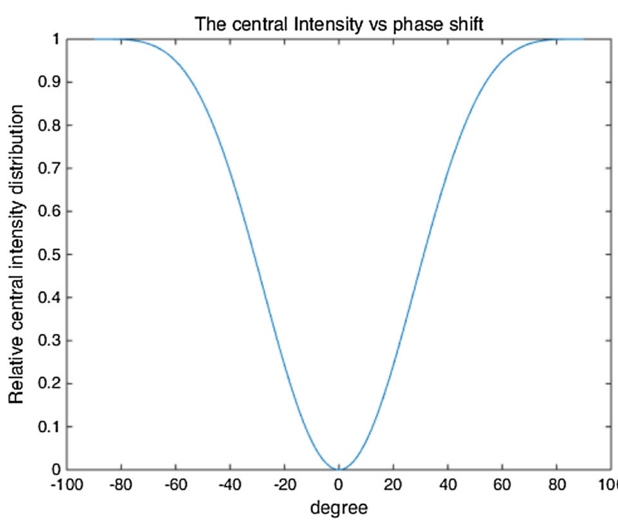


Fig. 4 The variation of the central Intensity with the half of phase shift δ

180° phase shift coating

In this section, we will design a thin film system to achieving $\delta_p - \delta_s = \pm\pi$, in other words $r_s + r_p = 0$.

The expressions of r_s and r_p can be written as [7]:

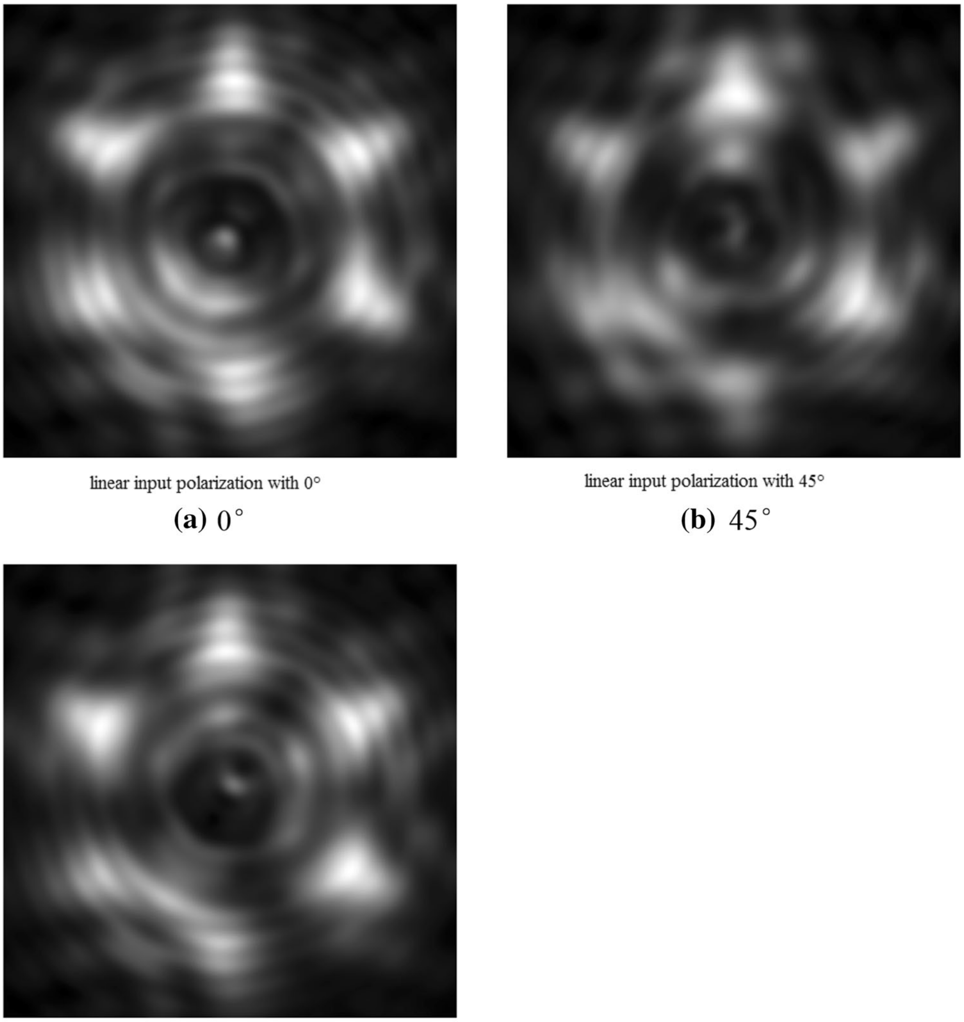
$$\begin{aligned} r_s &= \frac{\eta_s - Y_s}{\eta_s + Y_s} \\ r_p &= \frac{\eta_p - Y_p}{\eta_p + Y_p} \end{aligned} \tag{11}$$

The Eq. (12)

$$Y_s Y_p = n_0^2 \tag{12}$$

can be got by use of the Eq. (11).

By detail calculation, the result $Y_s Y_p$ for 2 k + 1 layer coatings and for 2 k layer coatings are got in the following Eq. (13):



(a) 0° **(b)** 45° **(c)** 90°

Fig. 5 Diffraction patterns of the CCR with the linear input polarization for the three orientations **a** 0°, **b** 45°, **c** 90°

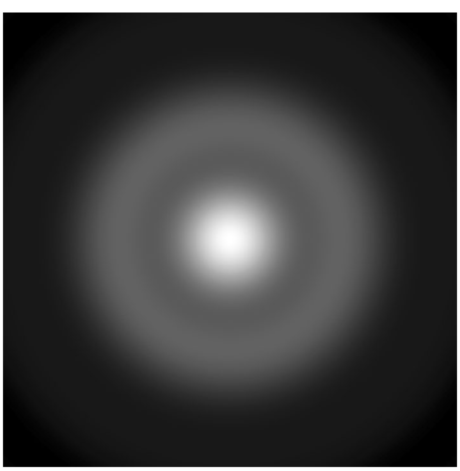


Fig. 6 Diffraction patterns of the CCR with a thin film satisfying $\delta_p - \delta_s = \pm\pi$

$$Y_s Y_p = \begin{cases} \dfrac{n_1^4 n_3^4 \cdots n_{2k+1}^4}{n_2^4 n_4^4 \cdots n_{2k}^4 n_s^2} (2k+1) \\ \dfrac{n_1^4 n_3^4 \cdots n_{2k-1}^4 n_s^2}{n_2^4 n_4^4 \cdots n_{2k}^4} (2k) \end{cases} \tag{13}$$

Equation (13) is got under the following conditions: (1) at a certain wavelength λ_0; (2) the phase thickness of each layer is $(2m+1)\frac{\pi}{2}$, that is

$$\delta_j = \frac{2\pi}{\lambda_0} n_j d_j \cos(\theta_j) = (2m+1)\frac{\pi}{2} \tag{14}$$

In the phase expression (14), n_j is the refractive index of jth layer and θ_j is the incidence angle of jth layer and note that it is not zero.

These coatings are most similar to antireflection coatings in the demands of the refractive index, while the thickness must satisfy the phase requirements. Thus we can always find some materials to satisfy the Eq. (13). Such as the three-layer coatings 1.52|1.38|2.13|1.90|1 and doule-layer coatings 1.52|1.38|1.70|1.

Conclusions

The six different polarization transform matrices emerging from the six paths through the CCR will result from six different elliptical polarization states and complicate the far-field diffraction pattern. One of the efficient ways to optimize the diffraction pattern is to change the phase shift of the orthogonal components of the reflection coefficient by coating on the three TIR surfaces. When the phase shift $\delta_p - \delta_s = \pm\pi$, the six polarization transform matrices are the same. Under this condition, we can get the Airy diffraction pattern at a certain wavelength in the far field no matter what the polarization state.

And the film structures to make the phase shift $\delta_p - \delta_s = \pm\pi$ are studied. These film must satisfy: (1) the phase thickness of each layer is $(2m+1)\frac{\pi}{2}$; (2) the refractive index are meet the condition of $\frac{n_1^2 n_3^2 \cdots n_{2k+1}^2}{n_2^2 n_4^2 \cdots n_{2k}^2 n_s} = n_0$ for 2 k + 1-layer and $\frac{n_1^2 n_3^2 \cdots n_{2k-1}^2 n_s}{n_2^2 n_4^2 \cdots n_{2k}^2} = n_0$.

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