

Research of the polarization aberration on Smith prism

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Abstract

Using the law of refraction and reflection of vector form to trace the light direction of propagation in the Smith prism. Be based on it, get the reflection phase shifter δ_s and δ_p on every planes, consequently obtain virtue matrix using Jones vector. The distribution of the polarization state of emergent light wave from Smith prism is analyzed when the light is vertical incidence and different azimuth, homochromatism and linearly polarized. The polarization aberration feature of Smith prism is obtained. At last, the detection experiment is done using the Stokes parameter measurement, and gets the consistent result with theory. The text lay the foundation for studying polarization aberration that effects on imaging quality and detection and correction of polarization aberration.

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1. Introduction

Smith prism is one of the reflecting lens commonly used in the optical instrument, and it makes the image of the object rotating, The two elliptical polarized lights on the image surface are got when the linearly polarized light penetrates it, and there are different vibration orientations, it will get different polarization states. Severely imaging quality of the optical system is caused by the polarization aberration, so it is very important to research on the polarization. There are some mathematical expression on the elliptically polarized light such as Jones matrix, Poincare Sphere Method and so on. In this paper, a method is got based on Jones matrix and the vector representation of the refraction and reflection law, the polarization state is analyzed when the linearly polarized light propagation through the Smith prism.

2. Analytic Geometry of Smith prism and tracing ray

It presents the direction in Fig.1. In the actual calculation, we only compute the unit vector .Based on the fact that the angle between two surfaces (not the ridge surface) of the Smith prism is 48° and the angle between the up-ridge (or down-ridge) and xoz surface is 66° , the prism is made by the K9 glass. By computing, we can get the unit normal vectors of every interface as following Table 1.

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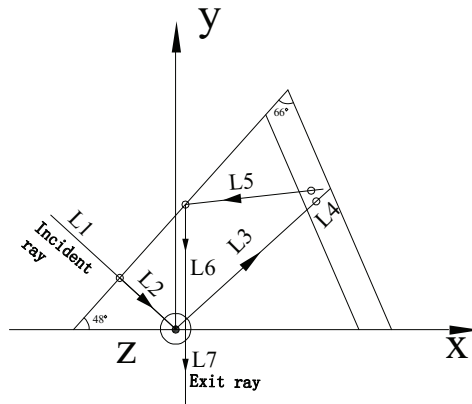


Fig.1. Coordinate system of Smith prism

Table1. The interface normal

The interface normal	
\vec{N}_1	(0, 0, -1)
\vec{N}_2	(-0.6459, -0.7071, 0.2876)
\vec{N}_3	(-0.6459, 0.7071, 0.2876)
\vec{N}_4	(0.7431, 0, 0.6691)

Under the situation that the light enter the prism rightly, i.e. $\vec{L}_1 = (0.7431, 0, 0.6691)$, we use the laws of the refraction and reflection in the presentation of vector^{[1][6]}and calculate the round in the len.

Table2. The vector of the reflecting light

	From up-ridge to down-ridge	From down-ridge to up-ridge
\vec{L}_1	(0.7431, 0, 0.6691)	(0.7431, 0, 0.6691)
\vec{L}_2	(0.7431, 0, 0.6691)	(0.7431, 0, 0.6691)
\vec{L}_3	(0.7431, 0, -0.6691)	(0.7431, 0, -0.6691)
\vec{L}_4	(-0.1257, 0.9510, -0.2823)	(-0.1257, -0.9510, -0.2823)
\vec{L}_5	(-0.9945, 0, 0.1045)	(-0.9945, 0, 0.1045)
\vec{L}_6	(0, 0, 1)	(0, 0, 1)

$$\vec{L}_7 \quad (0,0,1) \quad (0,0,1)$$

The unit normal to the plane defined by the normal of interface normal and output light ray is given by:

$$n_i = \frac{N_i \times L_i}{|N_i \times L_i|} \quad (i=1, 2, 3, 4, 5, 6) \tag{1}$$

Table3. The normal of every incidence plane

	From up-ridge to down-ridge	From down-ridge to up-ridge
\vec{n}_1	(0, -1, 0)	(0, -1, 0)
\vec{n}_2	(0, 1, 0)	(0, 1, 0)
\vec{n}_3	(0.6393, 0.2925, 0.7100)	(-0.6393, 0.2925, -0.7100)
\vec{n}_4	(0.0999, 0.2925, 0.9502)	(-0.0999, 0.2925, -0.9502)
\vec{n}_5	(0, 1, 0)	(0, 1, 0)
\vec{n}_6	(0, 1, 0)	(0, 1, 0)

3.The polarization state analysis of the emergent plane

The reflection Jones matrix^[2] when the light through the interface is given by

$$J_r = \begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix} \tag{2}$$

When light is the total reflection, the formula(1) Jones matrix^[2]is expressed by

$$J_r = \begin{bmatrix} \exp(i\delta_p) & 0 \\ 0 & \exp(i\delta_s) \end{bmatrix} \tag{3}$$

The refraction matrix^[2] when the light through the interface is:

$$J_t = \begin{bmatrix} t_p & 0 \\ 0 & t_s \end{bmatrix} \tag{4}$$

Because the normal of the every incidence planes of the Smith len is not coplanar, and for getting the effective matrix, we should obtain the rotation matrix to achieve coordinate conversion on the two incidence planes^[5]. then acquire the polarization state about the p-light and the s-light. In the course of ensuring the angle, it is must be confirmed whether the rotation is clockwise or anticlockwise form the incident plane normal of i to i+1. If the rotation is clockwise, the angle is negative angle, or else is positive angle .And the angle of the normal of incidence plane is:

$$\theta_{i,i+1} = \arccos \frac{[\mathbf{n}_i \cdot \mathbf{n}_{i+1}]}{\|\mathbf{n}_i\| \|\mathbf{n}_{i+1}\|} \tag{5}$$

According to the above formula, we will obtain the angles of incidence plane as the following Table 4.

Table 4. The angles of incidence plane

	From up-ridge to down-ridge	From down-ridge to up-ridge
θ_{12}	-180^0	-180^0
θ_{23}	72.8277^0	-72.8277^0
θ_{34}	-34.3446^0	34.3446^0
θ_{45}	72.8277^0	-72.8277^0
θ_{56}	0^0	0^0

The effective matrix for surface from i to i+1 is given by

$$T_i = R(\theta_{i,i+1})J_i, \tag{6}$$

During the formula,

$$R(\theta_{i,i+1}) = \begin{bmatrix} \cos(\theta_{i,i+1}) & \sin(\theta_{i,i+1}) \\ -\sin(\theta_{i,i+1}) & \cos(\theta_{i,i+1}) \end{bmatrix} \tag{7}$$

J_i is the reflection and the total refraction Jones matrix, the parameters are computed by the Fresnel in J_i . Using the total reflection and refraction Jones matrix, we will compute the effective matrix of the every planes of the Smith len, as the following Table 5.

Table5. The effective matrix of the every plane

	From up-ridge to down-ridge	From down-ridge to up-ridge
T_1	$\begin{bmatrix} -0.7936 & 0 \\ 0 & -0.7937 \end{bmatrix}$	$\begin{bmatrix} -0.7936 & 0 \\ 0 & -0.7937 \end{bmatrix}$
T_2	$\begin{bmatrix} -0.0517 - 0.2907i & 0.5530 - 0.7791i \\ 0.1672 + 0.9407i & 0.1709 - 0.2408i \end{bmatrix}$	$\begin{bmatrix} -0.0517 - 0.2907i & -0.5530 + 0.7791i \\ -0.1672 - 0.9407i & 0.1709 - 0.2408i \end{bmatrix}$
T_3	$\begin{bmatrix} -0.1250 - 0.8161i & -0.3355 + 0.4536i \\ -0.0854 - 0.5577i & 0.4910 - 0.6638i \end{bmatrix}$	$\begin{bmatrix} -0.1250 - 0.8161i & 0.3355 - 0.4536i \\ 0.0854 + 0.5577i & 0.4910 - 0.6638i \end{bmatrix}$
T_4	$\begin{bmatrix} -0.0447 - 0.2918i & 0.5682 - 0.7681i \\ 0.1446 + 0.9444i & 0.1756 - 0.2374i \end{bmatrix}$	$\begin{bmatrix} -0.0447 - 0.2918i & -0.5682 + 0.7681i \\ -0.1446 - 0.9444i & 0.1756 - 0.2374i \end{bmatrix}$
T_5	$\begin{bmatrix} -0.1750 - 0.9846i & 0 \\ 0 & 0.5788 - 0.8155i \end{bmatrix}$	$\begin{bmatrix} -0.1750 - 0.9846i & 0 \\ 0 & 0.5788 - 0.8155i \end{bmatrix}$
T_6	$\begin{bmatrix} 0.7937 & 0 \\ 0 & 0.7937 \end{bmatrix}$	$\begin{bmatrix} 0.7937 & 0 \\ 0 & 0.7937 \end{bmatrix}$

So the effective matrix of Smith len is: $T = T_6 \cdot T_5 \cdot T_4 \cdot T_3 \cdot T_2 \cdot T_1$ (8)

The effective matrix from up-ridge to down-ridge is:

$$T = \begin{bmatrix} 0.0522 + 0.3793i & -0.3005 - 0.3998i \\ 0.3005 + 0.3998i & 0.3498 + 0.1556i \end{bmatrix} \tag{9}$$

The effective matrix from down-ridge to up-ridge is:

$$T = \begin{bmatrix} 0.0522 + 0.3793i & 0.3005 + 0.3998i \\ -0.3005 - 0.3998i & 0.3498 + 0.1556i \end{bmatrix} \tag{10}$$

In allusion to the arbitrary orientation linearly polarized light $E_0 = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$. During the formula, θ is the included angle of the vibration orientation of the incidence linearly polarized light with x axis, so we will obtain the Jones matrix of output light is:

$$E = T * E_0 \tag{11}$$

The azimuth of elliptical polarized light is given by

$$\varphi = \frac{1}{2} \arctan\left(\frac{2E_{ox}E_{oy}}{E_{ox}^2 - E_{oy}^2} \cos(\delta)\right), \tag{12}$$

It is set $E_{ox} > E_{oy}$, so $-\pi/4 \leq \varphi \leq \pi/4$. When it does not have the limited situation, the φ should be from $-\pi/2$ to $\pi/2$.

Discussing through the following:

1) $E_{ox} > E_{oy}$

If $\delta \in (-\pi/2, \pi/2)$, $\begin{cases} \tan(2\varphi) > 0 \\ 0 \leq \varphi < \pi/2 \end{cases}$, it is got $0 \leq \varphi < \pi/4$;

If $\delta \in (-\pi/2, \pi) \cup \delta \in (-\pi, -\pi/2)$, $\begin{cases} \tan(2\varphi) < 0 \\ -\pi/2 < \varphi < 0 \end{cases}$, it is got $-\pi/4 \leq \varphi < 0$;

2) $E_{ox} < E_{oy}$

If $\delta \in (-\pi/2, \pi/2)$, $\begin{cases} \tan(2\varphi) < 0 \\ 0 \leq \varphi < \pi/2 \end{cases}$, we will get $\pi/4 < \varphi \leq \pi/2$;

If $\delta \in (\pi/2, \pi) \cup \delta \in (-\pi, -\pi/2)$, $\begin{cases} \tan(2\varphi) > 0 \\ -\pi/2 < \varphi < 0 \end{cases}$, get $-\pi/2 \leq \varphi \leq -\pi/4$;

According to the discussion, we get the regular of change of the azimuth with the vibration orientation of incidence linearly polarized light, and the range of the vibration orientation is $(-90^\circ, 90^\circ)$, as the Fig.2.

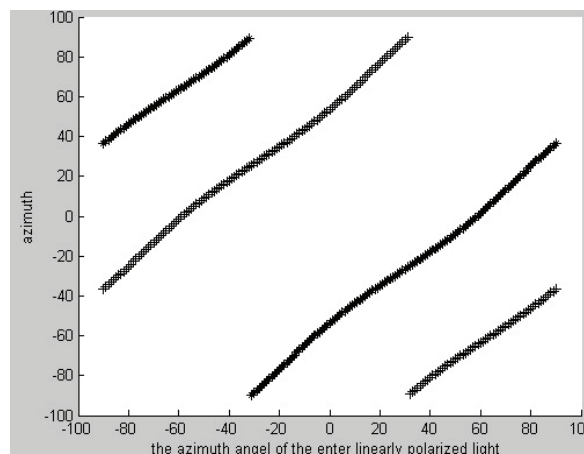


Fig.2 The diagram of curves of the azimuth angle

In the diagram, ‘black *’ expresses the change of the azimuth angle that the linearly polarize light incidencing from the up-ridge of the Smith len .while ‘black +’ expresses the change of the azimuth angle that the linearly polarized light incidencing from the down-ridge of the Smith len. The diagram make clear that the same azimuth of the linearly polarized light, we will obtain the two elliptically polarized light that there are different azimuths, but the included angle of the two azimuths is approximately $77^{\circ} \pm 4^{\circ}$, it has little change along with the vibration orientation.

The ellipticity is $\tan(\chi) = \frac{b}{a}$, and χ is ellipticity angle, and the relations of a 、 b and E_{ox} 、 E_{oy} is:

$$\begin{aligned} a^2 + b^2 &= E_{ox}^2 + E_{oy}^2 \\ \pm ab &= E_{ox} E_{oy} \sin \delta , \end{aligned} \tag{13}$$

δ is phase difference, and not division plus and minus. The variational ellipticity regular of the exit elliptically polarized light with the variational azimuth angle of the enter linearly polarized light at the range from -90° to 90° as Fig.3.

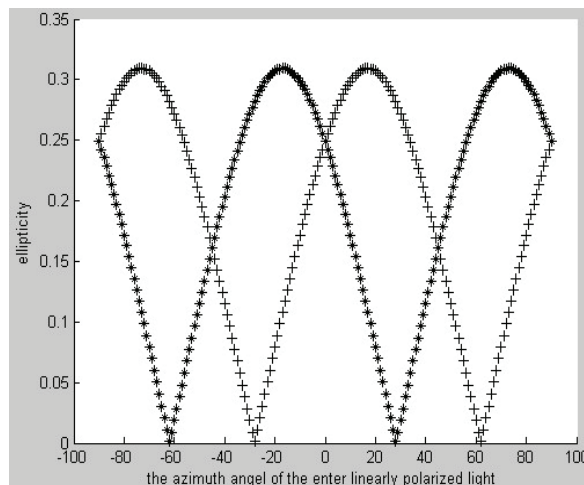


Fig.3. The diagram of curves of the ellipticity

In the diagram, ‘black *’ expresses the change of the ellipticity that the linearly polarize light incidencing from the up-ridge of the Smith len.while ‘black +’ expresses the change of the ellipticity that the linearly polarize light incidencing from the down-ridge of the Smith len. The diagram makes clear that the same azimuth of the linearly polarized light, we will obtain the two elliptical polarized light that there are different ellipticities, but there seems to be a certain phase with the vibration orientatin of incidencing linearly polarized light. But we will obtain the same ellipticity when the vibration orientation is -90° 、 -45° 、 0° 、 45° 、 90° .

The light intensity of the two elliptical polarized lights on the exitfacet is

$$I = E * E^+ , \tag{14}$$

E^+ is conjugation transposition matrix of E .The light intensity regular of the exit elliptical polarized lights with azimuth of the incidencing linearly polarized light is as Fig.4.

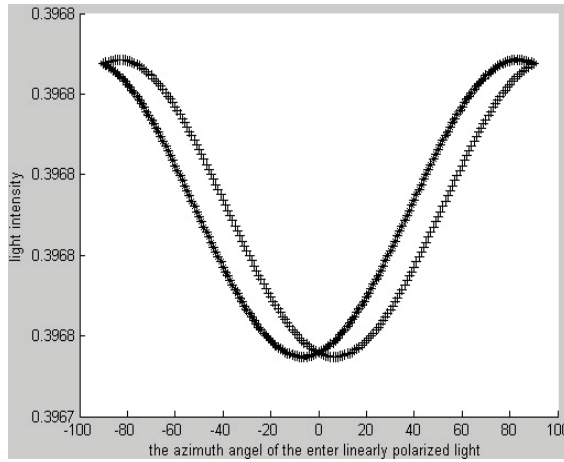


Fig.4 The diagram of curves of light intensity

In the diagram, ‘black +’ expresses the change of the light intensity that the linearly polarize light incidencing from the up-ridge of the Smith len. While ‘black *’ expresses the change of the light intensity that the linearly polarize light incidencing from the down-ridge of the Smith len. It makes clear that the light intensity of two elliptical polarized lights is same.

4.The result comparison between the experience and theory

Using the Stokes vector^[4] to prove the data got from the theory, the light path is as following Fig.5.

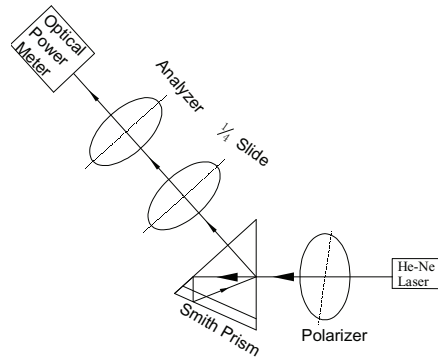


Fig.5 The light path of experiment

The comparison of the result of experiment and theory is as following Fig.6 and Fig.7.

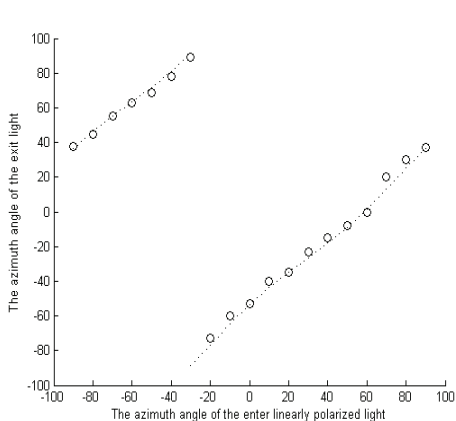


Fig.6(a) The change of azimuth from up-ridge

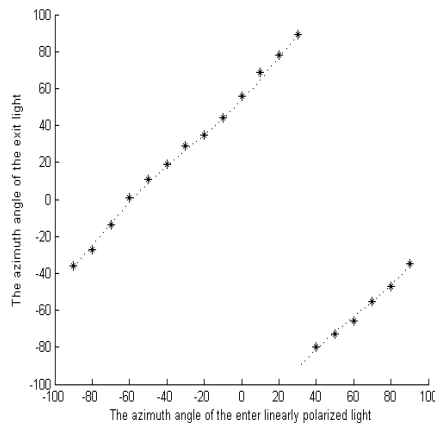


Fig.6(b) The change of azimuth from down-ridge

In the Fig.6(a) and Fig.6(b), ‘o’ and ‘*’ express the experimental azimuth value which the incidence light from up-ridge and down-ridge, while the plot of point of density express the theory value. The experiment and theory are consistent.

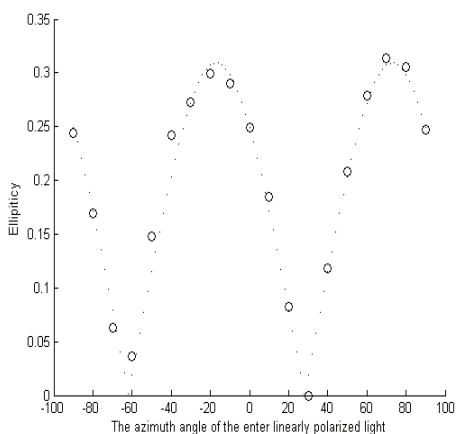


Fig.7(a) The change of ellipticity from up-ridge

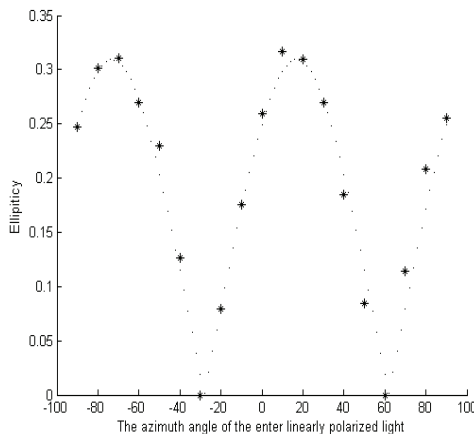


Fig.7(b) The change of ellipticity from up-ridge

In the Fig.7(a) and Fig.7(b), ‘o’ and ‘*’ express the experimental ellipticity value which the incidence light from up-ridge and down-ridge, while the plot of point of density express the theory value. It makes clear that experimental ellipticity value is close to the theoretical value.

5.Conclusion

The paper uses a set of theoretical calculations and experimental measurements, and according to a number of data and diagrams get the distribution of the polarization state which the linearly polarized light penetrates normally, and obtain the following conclusions: (1)When the linearly polarized light exits from up-ridge and down-ridge of Smith len, we will not get linearly polarized light, but two elliptically polarized light; (2) The elliptically polarized lights that exit from up-ridge and down-ridge has a certain phase, and we will obtain the same ellipticity when the vibration orientation of the linearly polarized light is -90° , -45° , 0° , 45° , 90° . (3) On the exit plane of Smith len will get the two elliptically polarized light that there are different azimuths, but the included angle of the two azimuths is within $77^\circ \pm 4^\circ$, it has change in a small range.

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