The Application of Thin Films Technique to Compensate Polarization Effects on Total Internal Reflection

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ABSTRACT

On total internal reflection (TIR) different phase shifts occur to the p- and s- states of polarization. Therefore the polarization state of the reflected light on TIR is generally elliptical polarized and differs from the state of polarization of the incoming wave. Only in cases where the entering wave has a pure p- or s- state relative to the plane of incidence these polarization states of the reflected wave are unchanged and hence are eigen-states.

In optical systems where the optical path is folded via TIR by using prisms, corner-cubes, retro-prisms, etc., the state of polarization of the exit wave will usually differ from the incoming one. In cases where the polarization state should be conserved, correcting retardation elements are applied to compensate the polarization effects on TIR. Instead of these bulk correcting elements, multilayer dielectric thin films stacks could be applied on the desired surfaces. This technique is introduced and discussed. An illustration of this method is represented on a retro - roof - prism of the type 45°-60° -60°.

INTRODUCTION

Many optical systems have reflecting elements which bend their optical axis such as dielectric or metallic mirrors, different types of prisms, beam-splitters and similar elements. The reason for this bending might be due to mechanical constrains, combining or dividing different optical channels, image conversion, folding in laser cavities etc. The angle of incidence of the traversing radiation at the reflecting surface is usually an oblique angle hence it might introduce polarization effects, namely, the state of polarization of the reflected wave will not be the same as that of the incident one. This is due to the fact that generally on reflection at non-normal incidence, the reflectances (R_p, R_s) and the phase shifts (δ_p, δ_s) for the p- and the s- states of polarization respectively are not the same. When the reflecting boundary is between two dielectric media, R_p differs from R_s, however the difference between the phase shifts is either 0° or 180°. When total internal reflection occurs, the reflectances R_p and R_s are the same (100%) but the phase shifts are different and usually are neither 0° nor 180°. Therefore, the effect of TIR on polarized light is as a retarder plate. In case where the reflecting boundary is between a dielectric and metallic (absorbing) media, TIR does not occur and generally, both the reflectances R_p and R_s and the phase shifts δ_p and δ_s will be different. When optical thin films (single or multilayer) are applied to the reflecting boundary, R_p, R_s, δ_p, and δ_s will be changed. Accordingly, optical thin films could be used as polarizing filters to control the polarization state of the traversing waves in an optical system.

In the literature there are reports on thin film retarders[1-5]. It is desired to have such retarders due to their low cost relative to expensive crystalline plates such as quarter-wave etc. and therefore they could replace

them. A competitive thin film retarder should have the following characteristics:
- High reflectance for both states of polarization (R_p and R_s).
- Both reflectances should be equal ($R_p = R_s$).
- The retardation should have a specified value.
- For a small range of angles of incidence and wavelengths around the specified values, both reflectances should be high and equal and the retardation should remain constant.

Apfel[1] introduced a graphical method to design multilayer phase retarder mirrors. To illustrate this method he used a dielectric periodic quarter-wave stack to achieve high reflectances with the addition of a few more layers to get a desired retardance. He used his graphical method to examine Southwell's 90° retarder which is based on four non-periodic dielectric layers on top of a silver mirror[2]. In Ref. 3, Apfel studied the retardance of periodic multilayer dielectric reflectors, it was found that the obtainable retardance is less than 30°. Thin film retarders based on TIR were studied by Korneev[4], Apfel[3] and Spiller[5]. Korneev applied coatings consisting of up to three dielectric layers to the reflecting surfaces of corner prisms to achieve a reflectance of 100% (TIR) with a given retardance. This coating substituted silver coatings applied to the relevant surfaces. Hence the pre-specified retardance for the dielectric coatings was the same value which was obtained by the silver coatings. Apfel introduced a graphical method to analyze thin film TIR phase retarders. Spiller introduced TIR single and multilayer thin film retarders having a 90° phase shift to substitute quarter-wave plates.

In this work TIR multilayer thin film retarders to conserve the polarization state of the radiation in optical systems are represented, illustrated and discussed.

BASIC THEORY, EXAMPLE AND DISCUSSION

On reflection at an oblique angle of incidence the polarization state generally will not be conserved, namely the incident and the reflected light will have different states of polarization, due to non equal reflectances and/or phase shifts of the p- and s- polarization components. In the following the polarization states will be calculated and described by Jones calculus, Stokes parameters and Poincaré sphere using Dirac notation and according to Simmons and Guttmann[6]. The axis basis where the different polarization states are described is composed of two ortho-normal P-states namely, $| P_x >$ and $| P_y >$ where $| P >$ is a linear polarization state and are given by:

$$| P_x > = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad | P_y > = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

The plane x-y is chosen arbitrarily. A general state $| E >$ is represented as:

$$| E > = a \exp(-i\phi/2) \, | P_x > + b \exp(i\phi/2) \, | P_y > \tag{2}$$

where a and b are expressing either amplitudes or relative amplitudes of the different components depending on the normalization namely, $a^2 + b^2 = I$ (I is the beam intensity) or $a^2 + b^2 = 1$ respectively and ϕ is the phase shift $(\phi_y - \phi_x)$[7]. A polarization state is unchanged by adding a constant phase to both components, therefore:

$$| E' > = \exp(i\psi)\ (I)\ | E > \tag{3}$$

where (I) is a unit matrix. The polarization state will also be conserved on multiplying both components with a real constant C, however the intensity will be changed by C^2. When a beam at a general polarization state $| E >$ as given by Eq.(2) will be reflected at an oblique angle of incidence, the reflected beam will have a new state $| E' >$. If the polarization states with respect to the plane of incidence is the same x - y plane where the polarization states are described, then:

$$| E' > = \begin{pmatrix} r_p \exp(i\delta_p) & 0 \\ 0 & r_s \exp(i\delta_s) \end{pmatrix} | E > \tag{4}$$

Otherwise:

$$| E' > = R^{-1}(\Theta) \begin{pmatrix} r_p \exp(i\delta_p) & 0 \\ 0 & r_s \exp(i\delta_s) \end{pmatrix} R(\Theta)\ | E > \tag{5}$$

where Θ is the angle between the x - y plane and the incidence plane, and R is the orthogonal matrix that rotates the axis system with the desired angle. $R(\Theta)$ and $R^{-1}(\Theta)$ are:

$$R(\Theta) = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \quad ; \quad R^{-1}(\Theta) = \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \tag{6}$$

The polarization operator which represents the reflection at an oblique angle of incidence when the axis system coincides with the system defined by the plane of incidence and the normal to it, is according to Eq.(5):

$$T = \begin{pmatrix} r_p \exp(i\delta_p) & 0 \\ 0 & r_s \exp(i\delta_s) \end{pmatrix} \tag{7}$$

When the axis systems are not the same T is:

$$T = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} r_p \exp(i\delta_p) & 0 \\ 0 & r_s \exp(i\delta_s) \end{pmatrix} \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \tag{8}$$

where Θ is the angle between the two systems. The operator T in Eq.(7) can be written as:

$$T = \sqrt{r_p^2 + r_s^2} \exp[i(\delta_p + \delta_s)/2] \begin{pmatrix} \exp[-i(\delta_s - \delta_p)/2] & 0 \\ 0 & \exp[i(\delta_s - \delta_p)/2] \end{pmatrix} \begin{pmatrix} \frac{r_p}{\sqrt{r_p^2+r_s^2}} & 0 \\ 0 & \frac{r_s}{\sqrt{r_p^2+r_s^2}} \end{pmatrix}$$

The term $C = \sqrt{r_p^2 + r_s^2} \exp[i(\delta_p + \delta_s)/2]$ is a constant multiplier to both polarization components hence not affecting the polarization state (it affects only the total reflected intensity) and hence it could be omitted. Therefore the operator T could be written as :

$$T = \begin{pmatrix} \exp(-i\delta/2) & 0 \\ 0 & \exp(i\delta/2) \end{pmatrix} \begin{pmatrix} \frac{r_p}{|C|} & 0 \\ 0 & \frac{r_s}{|C|} \end{pmatrix} \tag{9}$$

where $\delta = \delta_s - \delta_p$ is the retardance. When eq.(8) is applicable then:

$$T' = R^{-1}(\theta)TR(\theta) \tag{10}$$

where R is the rotation matrix to rotate the axis system. The matrix:

$$D = \begin{pmatrix} \frac{r_p}{|C|} & 0 \\ 0 & \frac{r_s}{|C|} \end{pmatrix} \tag{11}$$

is an operator which represents a dichroic linear polarizer which absorbs unisotropically in orthogonal directions (Ref. 6 page 69). The matrix:

$$C = \begin{pmatrix} \exp(-i\delta/2) & 0 \\ 0 & \exp(i\delta/2) \end{pmatrix} \tag{12}$$

is an operator representing a retarder. Therefore, a reflection at non-normal incidence, from the point of view of polarization, represents generally a dichroic linear polarizer followed by a phase retarder (or vice versa since both operators are diagonal matrices and therefore they commute). However, when the entrance wave is either p- or s- state with respect to the plane of incidence there will be no change in the polarization state since the reflection in this case represents a multiplication of the original state by a constant. Hence, a p- or s- state relative to the plane of incidence is an eigen-value of the reflection operator. When on reflection both reflectances are equal ($R_p = R_s$) then the effect on the polarization state is as a phase retarder only, but generally with reduced intensity. However, on TIR phase retardance occurs without intensity attenuation. The effect of TIR is therefore represented by the operator C as given in Eq.(12) where δ is $\delta_s - \delta_p$ (the phase shift δ is as in Ref.(6) and not $\delta_p - \delta_s$), and the frame of reference of the axis system is as defined by the plane of incidence. For a different axis system the operator is accordingly $R^{-1}(\theta)CR(\theta)$, where θ is the angle between the two systems.

The compensation of the retardance which occurs on TIR is achieved either by using an additional appropriate retarder plate oriented in the desired direction or by applying dielectric thin films to the reflective surface causing equal phase shifts to p- and s- polarization states. The phase shifts on TIR are calculated from Fresnel equations[5]. These equations are:

$$r_p = \frac{n_0 \cos\alpha - n\cos\alpha_0}{n_0\cos\alpha + n\cos\alpha_0} \quad ; \quad r_s = \frac{n_0\cos\alpha_0 - n\cos\alpha}{n_0\cos\alpha_0 + n\cos\alpha} \tag{13}$$

where α_0 is the incident angle and α is the angle which the light propagates in the second medium. In the present case this angle is imaginary, since on TIR $(n_0/n)\sin\alpha_0 = \sin\alpha > 1$.

Example:
For entrance medium n = 1.507 ; Exit medium n = 1 (Air) and $\alpha_0 = 45°$, $\delta = 141.88°$.
For $\alpha_0 = 60°$, $\delta = 139.29°$

We designed dielectric multilayer thin film stacks for boundaries where TIR occurs. The low refractive index layers were restricted to ensure real propagation angles in these layers, consequently TIR occurs only at the last boundary. We did not consider thin layers with imaginary propagation angles to avoid evanescent wave coupling or tunneling to the next high index layers. The stacks were designed for 45° and 60° angles of incidence in an entrance medium with refractive index 1.507 and air as the exit medium (n = 1). These designs are for application on a 45° retro-prism where one of the 45° faces is a roof. This prism, and the light propagation in it is shown in Figure 1. Here, the light is incident first on the 45° face and then on the roof faces, however the direction could be reversed.

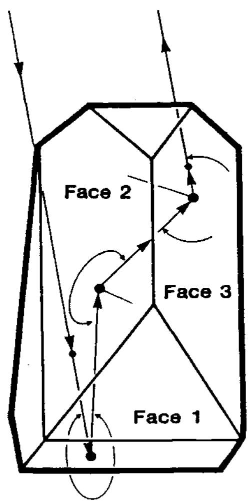

Figure 1. *The 45° retro-roof-prism and the light propagation in it*

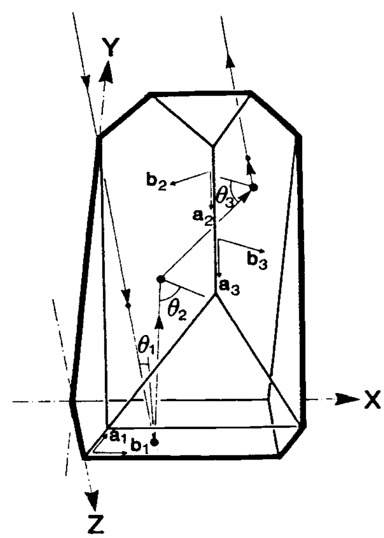

Figure 2. *The description of the light path in the x-y-z axis system*

The prism with n=1.507 is surrounded by air, hence at all the faces TIR occurs. Since the reflections are at different planes of incidence therefore the exit light (without compensation) will be elliptical polarized and at a different state with respect to the incident wave. In order to calculate the polarization state after each reflection, the plane of incidence with respect to an axis system where the polarization states are described should be calculated as well as the angle of incidence. In Figure 2 the axis system, the reflecting faces and the light path are shown. In this figure, Face 1 is parallel to the x-axis and inclined at 45° to the x-y plane. The angle between Faces 2 and 3 which produce the roof is 90°. The plane of the roof which is defined as the base of the prism constructed by the Faces 2 and 3 is inclined also at 45° to the x-y plane. The light propagates parallel to the z-axis, reflected from Face 1, followed by reflections from Faces 2 and 3 and exits in the -z direction. The orientation of the three faces are defined by their normal $\vec{N_i}$, $i = 1,2,3$. The directions of the $\vec{N_i}$ are calculated by the vectorial product $\vec{a_i} \times \vec{b_i} = \vec{N_i}$, where $\vec{a_i}$ and $\vec{b_i}$ are two co-planar unit vectors on the i-th Face. The vectors $\vec{N_i}$ are then normalized. The vectors $\vec{a_i}$ and $\vec{b_i}$ are also shown on Fig. 2. The vectors $\vec{a_i}$, $\vec{b_i}$ and accordingly $\vec{N_i}$ are:

$$\vec{a_1} = 1/\sqrt{2}(\vec{j}+\vec{k}) \quad ; \quad \vec{b_1} = \vec{i} \quad ; \quad \vec{N_1} = 1/\sqrt{2}(\vec{j}-\vec{k}) \tag{14}$$

where $\vec{i}, \vec{j}$ and $\vec{k}$ are unit vectors along the axes. The vector $\vec{a_2}$ is equal to $\vec{a_3}$ and is:

$$\vec{a_2} = \vec{a_3} = 1/\sqrt{2}(-\vec{j}+\vec{k}) \tag{15}$$

To calculate $\vec{b_2}$ and $\vec{b_3}$ we rotate the x-y-z system around the x-axis by 45° and therefore $\vec{a_2}$ and $\vec{a_3}$ are parallel to the z' axis as shown in Figure 3. Accordingly, $\vec{a'_2} = \vec{k'}$, or:

$$\vec{a'_2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

In the new system $\vec{b'_2}$ will be:

$$\vec{b'_2} = 1/\sqrt{2}(-\vec{i'} - \vec{j'})$$

Therefore in the old system $\vec{b_2}$ is:

$$\vec{b_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$$

or:

$$\vec{b_2} = -1/2(\sqrt{2}\,\vec{i}+\vec{j}+\vec{k}) \tag{16}$$

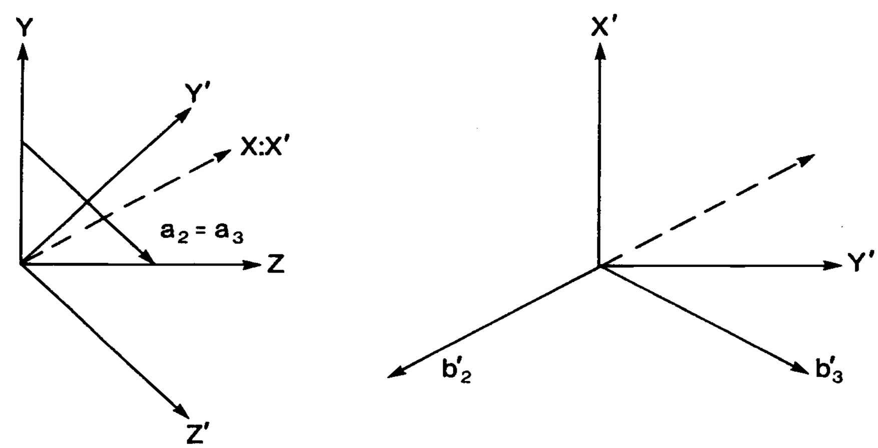

Figure 3. *The description of the vectors in the axis systems*

Similarly, $\vec{b_3'}$ in the new system is: $\vec{b_3'} = 1/\sqrt{2}(-\vec{i'} + \vec{j'})$

Hence, in the old system it is:

$$\vec{b_3} = 1/2(-\sqrt{2}\,\vec{i} + \vec{j} + \vec{k}) \tag{17}$$

From $\vec{a_2} = \vec{a_3}$, $\vec{b_2}$ and $\vec{b_3}$, $\vec{N_2}$ and $\vec{N_3}$ can be calculated according to $\vec{N} = \vec{a} \times \vec{b}$ and normalized. They are:

$$\vec{N_2} = 1/2(\sqrt{2}\,\vec{i} - \vec{j} - \vec{k}) \quad ; \quad \vec{N_3} = -1/2(\sqrt{2}\,\vec{i} + \vec{j} + \vec{k}) \tag{18}$$

The angle and plane of incidence for each interface can be calculated according to[8]:

$$\vec{P'} = \vec{P} - 2(\vec{P} \cdot \vec{N})\vec{N} \tag{19}$$

Or in matrix notation as:

$$\begin{pmatrix} P_x' \\ P_y' \\ P_z' \end{pmatrix} = \begin{pmatrix} 1 - 2N_x^2 & -2N_xN_y & -2N_xN_z \\ -2N_yN_x & 1 - 2N_y^2 & -2N_yN_z \\ -2N_zN_x & -2N_zN_y & 1 - 2N_z^2 \end{pmatrix} \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} \tag{19a}$$

where:
- $\vec{P}$ is a unit vector along the incident beam.
- $\vec{P'}$ is the vector along the reflected beam and therefore after normalization $\vec{P_i'} = \vec{P}_{i+1}$
- $\vec{N}$ is the vector normal to the reflecting interface as calculated before.

Since $\vec{P}$ and $\vec{N}$ are unit vectors, hence:

$$\cos\alpha = \vec{P} \cdot \vec{N} \tag{20}$$

where α is the angle of incidence.

The plane of incidence is defined by the vector which is normal to it and is given by:

$$\vec{e_i} = \vec{P_i} \times \vec{N_i} \quad or \quad \vec{e_i} = \vec{P_i} \times \vec{P_i'} \tag{21}$$

From Eqs.(18-21) and for a given $\vec{P_1}$ the values for $\vec{P_i}$, and $\vec{e_i}$ can be calculated, they are:

$$\begin{array}{lll} \vec{P_1} = \vec{k}\ ; & \vec{P_2} = \vec{j}\ ; & \vec{P_3} = 1/2(\sqrt{2}\,\vec{i} + \vec{j} - \vec{k}) \\ \alpha_1 = 45^\circ\ ; & \alpha_2 = 60^\circ\ ; & \alpha_3 = 60^\circ \\ \vec{e_1} = \vec{i}\ ; & \vec{e_2} = \sqrt{1/3}\,\vec{i} + \sqrt{2/3}\,\vec{k}\ ; & \vec{e_3} = \sqrt{1/3}\,\vec{i} - \sqrt{2/3}\,\vec{j} \end{array}$$

The vectors $\vec{e_i}$ are perpendicular to the planes of incidence therefore they are parallel to the s component of the polarization states in the eigen-system of the relevant reflectors. The angles between the planes of incidence are:

$$\begin{aligned} \cos(e_1, e_2) &= \sqrt{1/3}; \quad \angle(e_1, e_2) = 54.7^\circ \\ \cos(e_2, e_3) &= 1/3; \quad \angle(e_2, e_3) = 70.5^\circ \\ \cos(e_1, e_3) &= -\sqrt{1/3}; \quad \angle(e_1, e_3) = 125.3^\circ \end{aligned} \tag{22}$$

At this stage the polarization states can be determined, since we already calculated the retardances and the planes of incidence for a wave propagating parallel to the z axis. Let the polarization states be described in the axis system as defined by the eigen-system of Face 1 (Fig. 2) namely, $| P_x >$ (p state) is parallel to the y-axis and $| P_y >$ (s state) is parallel to the x-axis. If the incident wave is either a $| P_x >$ or a $| P_y >$ state, the state of the reflected wave from Face 1 will be conserved. For the general case the polarization operator of the prism is as follows:

$$O = R_{31}^{-1} C_3 R_{31} R_{21}^{-1} C_2 R_{21} C_1$$

or:

$$O = R_{31}^{-1} C_3 R_{32} C_2 R_{21} C_1$$

where C_i is the retardance operator as given in Eq.(12) and R_{ij} is the orthogonal matrix which rotates the axis system from the eigen-system of Face j to the eigen-system of Face i. Using Eqs.(22) and (6) and the phase shift values as given in the example, the operator O can be calculated. If the state of the incident wave is:

$$| E_0 > = a \exp(-i\phi_0/2) \mid P_x > + b \exp(i\phi_0/2) \mid P_y >$$

then the state of the exit wave will be:

$$| E' > = O \mid E_0 > = O \begin{pmatrix} a \exp(-i\phi_0/2) \\ b \exp(i\phi_0/2) \end{pmatrix} \tag{23}$$

If $| E_0 >$ is a $| P_x >$ state then the states after each reflection will be as follows: After the reflection at Face 1 the polarization state is conserved, since $| P_x >$ is an eigen-state. After reflection at Face 2 the state is $| E_2 >$,

$$| E_2 > = R_{21}^{-1} C_2 R_{21} \mid P_x >$$

or:

$$| E_2 > = \frac{1}{3}\begin{pmatrix} 1 & -\sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} e^{-69.65^\circ i} & 0 \\ 0 & e^{69.65^\circ i} \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$| E_2 > = \begin{pmatrix} 0.4681e^{65.935^\circ i} \\ 0.8837e^{-65.935^\circ i} \end{pmatrix} = \begin{pmatrix} \cos 62.09^\circ & e^{65.935^\circ i} \\ \sin 62.09^\circ & e^{-65.935^\circ i} \end{pmatrix} \quad (24)$$

Similarly, after the reflection at Face 3 $| E_3 >$ will be:

$$| E_3 > = \frac{1}{3}\begin{pmatrix} -1 & -\sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} e^{-69.65^\circ i} & 0 \\ 0 & e^{69.65^\circ i} \end{pmatrix}\begin{pmatrix} -1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{pmatrix} | E_2 >$$

or:

$$| E_3 > = \begin{pmatrix} 0.834e^{-82.42^\circ i} \\ 0.5518e^{82.42^\circ i} \end{pmatrix} = \begin{pmatrix} \cos 33.49^\circ & e^{-82.42^\circ i} \\ \sin 33.49^\circ & e^{82.42^\circ i} \end{pmatrix} \quad (25)$$

The state $| E_3 >$ is the state of the exit wave and in order to conserve the polarization state, an external retarder should be applied with retardance δ and in orientation θ by solving the equations:

$$\exp(i\epsilon) \mid P_x > = M \mid E_3 >$$

where M is:

$$M = R^{-1}(\theta)C(\delta)R(\theta)$$

and ϵ is a free phase parameter that does not affect the polarization state. The values of δ and θ can also be determined geometrically, by using Stokes parameters and Poincaré Sphere[6]. The Stokes parameters P of a general state $| E >$ are:

$$| E > = \begin{pmatrix} \cos\psi \exp(-i\phi/2) \\ \sin\psi \exp(i\phi/2) \end{pmatrix} \implies P = \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \cos(2\psi) \\ \sin(2\psi)\cos\phi \\ \sin(2\psi)\sin\phi \end{pmatrix}$$

In our case, the Stokes parameters for $| E_1 > = | E_0 > = | P_x >$, $| E_2 >$ and $| E_3 >$ as given in Eqs.(24,25) are:

$$| E_1 > = | E_0 > \rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \; ; \quad | E_2 > \rightarrow \begin{pmatrix} 1 \\ -0.5618 \\ -0.5522 \\ -0.616 \end{pmatrix} \; ; \quad | E_3 > \rightarrow \begin{pmatrix} 1 \\ 0.3911 \\ -0.8883 \\ 0.2407 \end{pmatrix} \quad (26)$$

In Figure 4 these states are described on Poincaré sphere. The operation of a retarder is described on Poincaré sphere by a rotation with the retardance angle around an axis which is a diameter on the equatorial plane. The angle of this axis with the P_1 axis is twice of the orientation angle of the retader. The initial state is $| E_0 >=| P_x >$. The rotation axis due to TIR from Face 1 (Fig. 1) is the P_1 axis. Since the polarization state is on this axis, hence no retardance effect is occured hence $| E_1 >=| E_0 >$. The angle of the P_1 axis with the rotation axis A_2 due to TIR from Face 2 is $2 \times \angle(e_1.e_2)$ and according Eq.(22) it is 109.4° . The retardance angle is 139.29° . Accordingly, $| E_1 >$ is transfered to $| E_2 >$. Similarly, the rotation axis A_3 is according to Eq.(22) at angle of 250.6° to the P_1 axis and the retardance angle is also 139.29° . Due to the rotation, $| E_2 >$ is transfered to $| E_3 >$. In Figure 5 the orientation of the external retarder and the amount of the retardance to transfer the polarization state of the exit wave $| E_3 >$ to $| P_x >$ is shown. The rotation axis A is defined by the diameter on the equatorial plane which is normal to the cord obtained by the $| P_x >$ point and the projection of $| E_3 >$ on the equatorial plane. On rotation around A the state $| E_3 >$ will be transfered to $| P_x >$ as required.

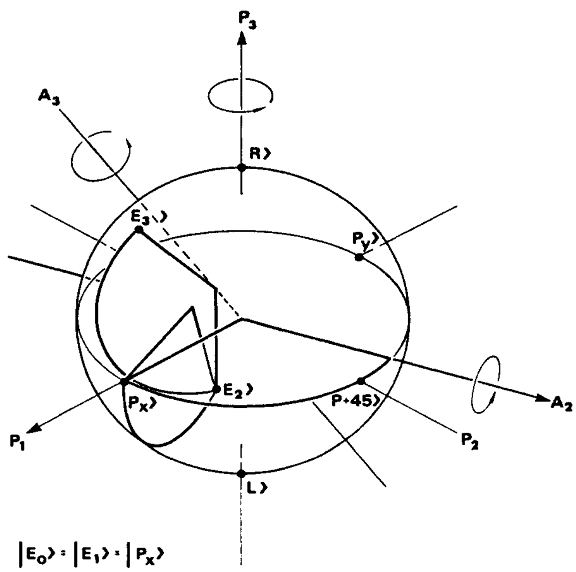

Figure 4. *The description of the different states on Poincaré sphere*

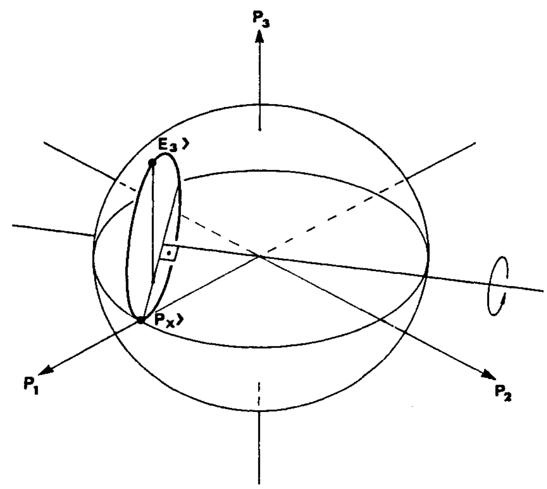

Figure 5. *The description of the external retarder and its orientation on Poincaré sphere*

The compensation can also be achieved by the application of thin films to the faces where the TIR occurs. However, in the case where the incident state is either $| P_x >$ or $| P_y >$, Face 1 should not be coated since they are eigen-states and only the faces of the roof should be coated with the same coating to compensate the retardance which occurs at 60° angle of incidence. We designed two stacks of dielectric multilayer coatings for the roof faces and for Face 1 where the refractive indices of the entrance and exit media are 1.507 and 1 respectively. The coating for 60° angle of incidence consists of four dielectric layers and the retardance as a function of normalized wave number g is shown in Figure 6. The parameter g is defined as λ_0/λ where λ is the desired wavelength and λ_0 is an arbitrarily chosen reference wavelength where the optical thicknesses of the layers are related. This coating design is effective at least over the range $0.965 \le g \le 1.03$.

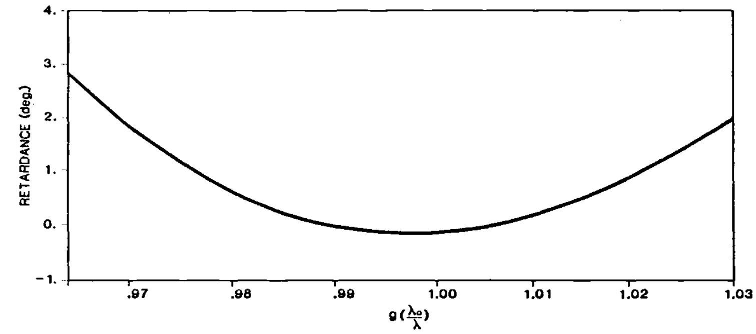

Figure 6. *The retardance as a function of g for a thin film stack designed for 60° angle of incidence*

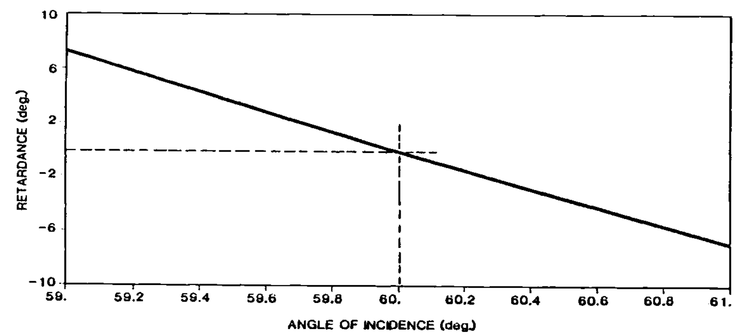

Figure 7. *The effect of angle of incidence on the retardance at* λ_0 *for the 60° designed stack*

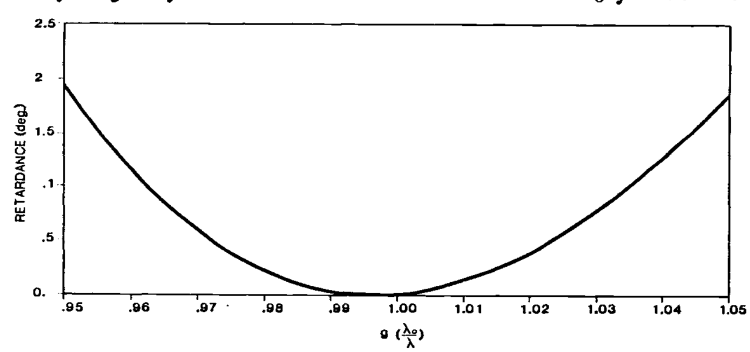

Figure 8. *The retardance as a function of g for a thin film stack designed for 45° angle of incidence*

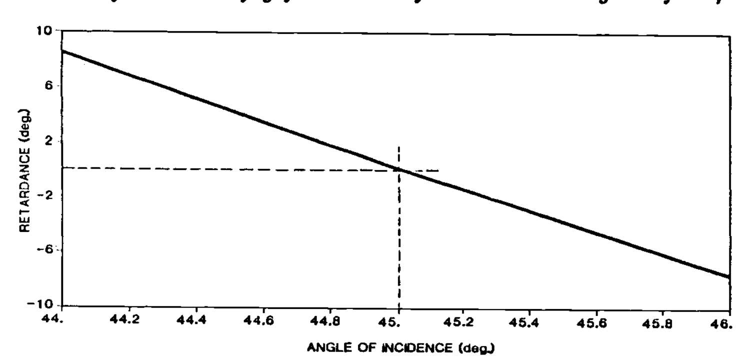

Figure 9. *The effect of angle of incidence on the retardance at* λ_0 *for the 45° designed stack*

If for example the required compensation is for $\lambda = \lambda_0 = 1000\ nm$, then the effective wavelength range is 970 - 1036 nm. The effect of the angle of incidence on the performance of the coating at λ_0 is shown in Figure 7. The coating is effective at least 1/2 degree around 60° which is the design angle. Similar curves are shown in Figures 8 and 9 of a thin film design consisting also of four layers for 45° angle of incidence, suitable for Face 1. This coating should be provided in a case where the initial state is neither p nor s with respect to the plane of incidence.

CONCLUSIONS

In this work, the application of dielectric thin films to conserve polarization states on TIR was introduced. This method can substitute expensive birefringent plates. The use of such plates causes losses in light transmission due to Fresnel reflections from the plate surfaces. For first order retarder plates to be used for high energy lasers, air spaced plates should be applied and hence there are four interfaces. Even in a case where these intefaces are antireflection coated, yet, there are losses of at least 0.5%. However, in the thin film case no such losses exist. Moreover, there are cases where birefringent plates are not applicable and only thin films can be used. These cases occur in optical systems where the light propagates in two opposite directions through the same optical elements (as in laser gyro systems). In such cases a retarder plate for one direction generally will not compensate properly the phase of the wave in the opposite direction, since polarizing operators do not commute.

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7. Note: The phase retardation δ in Ref.(6) is defined as $\delta_s - \delta_p$, however in the other Refs. it is defined as $\delta_p - \delta_s$.

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